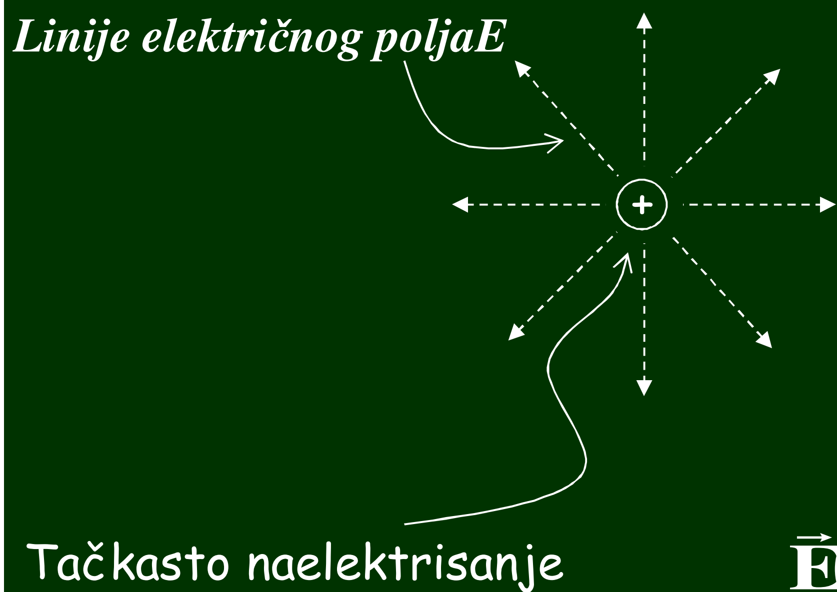
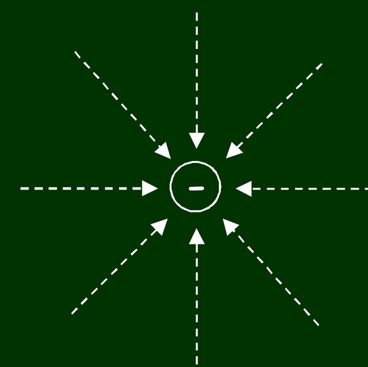
MAKSVELOVE JEDNAČINE

I.II. Gausov zakon

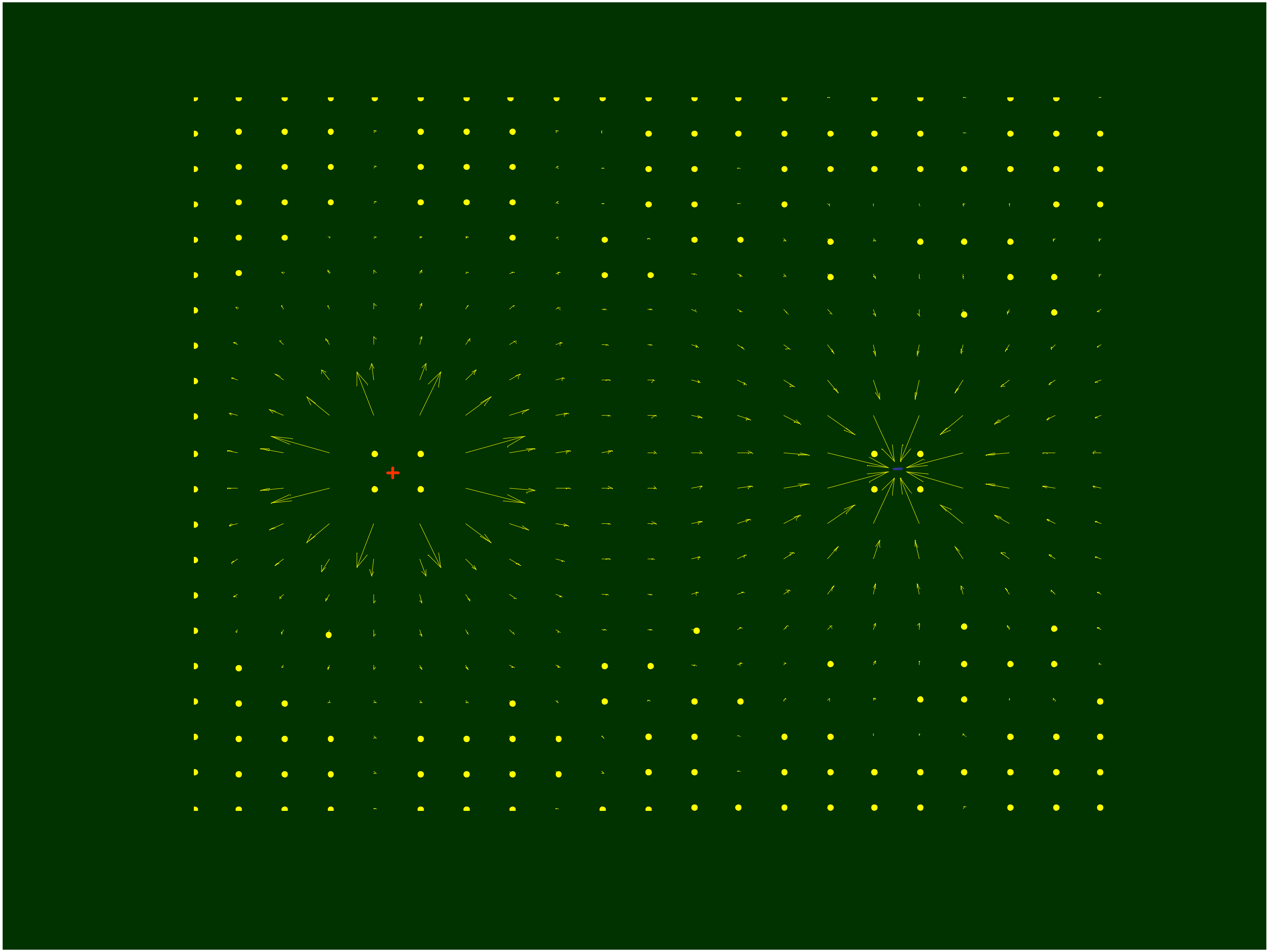
Daje vezu između raspodele naelektrisanja i električnog polja

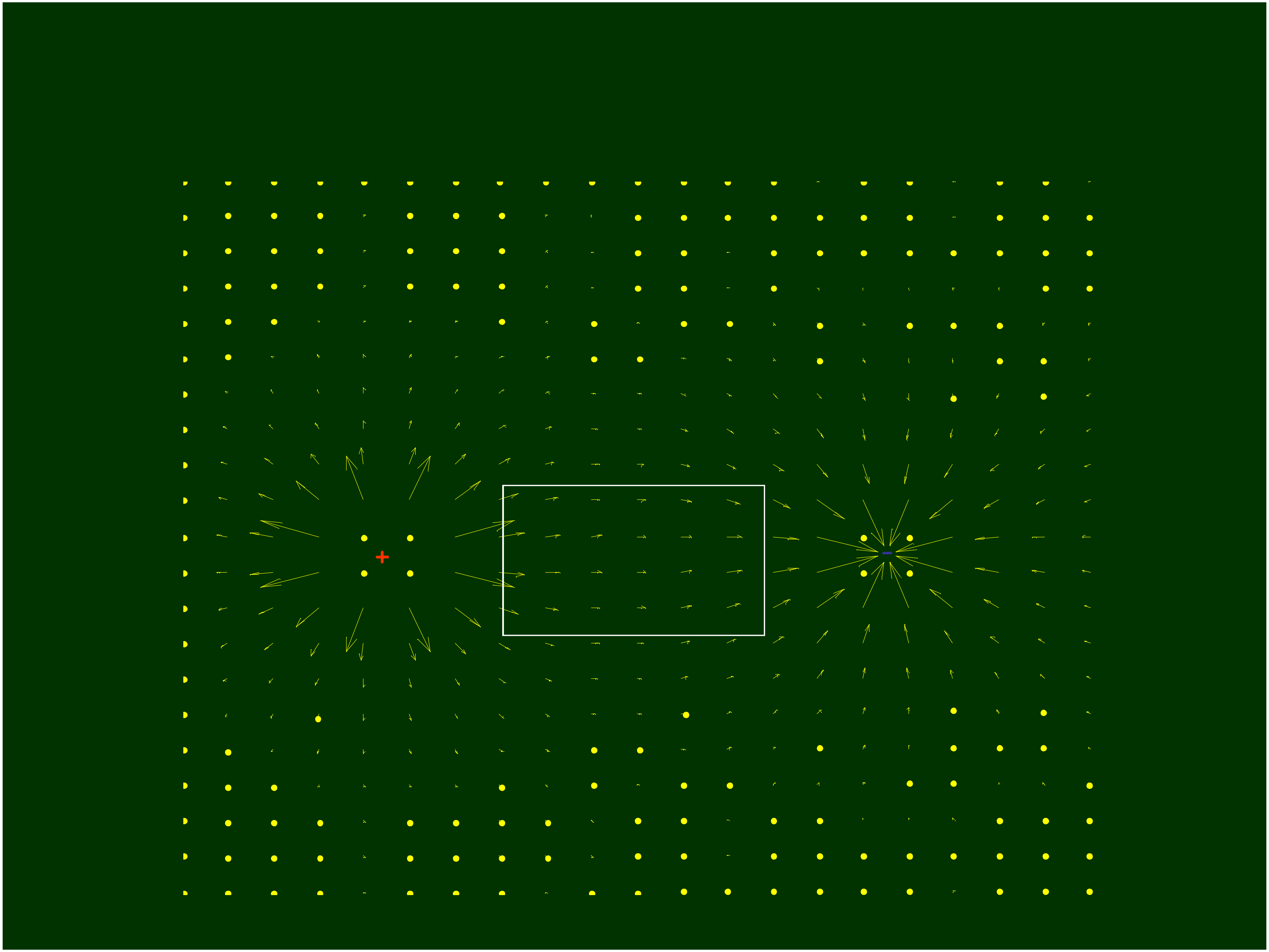


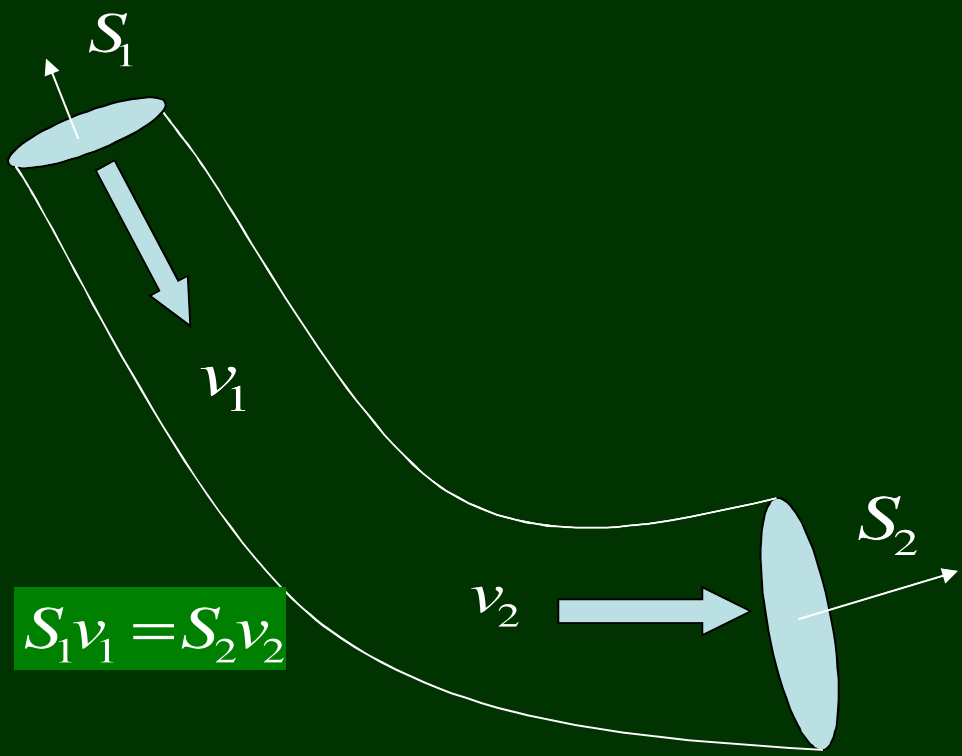


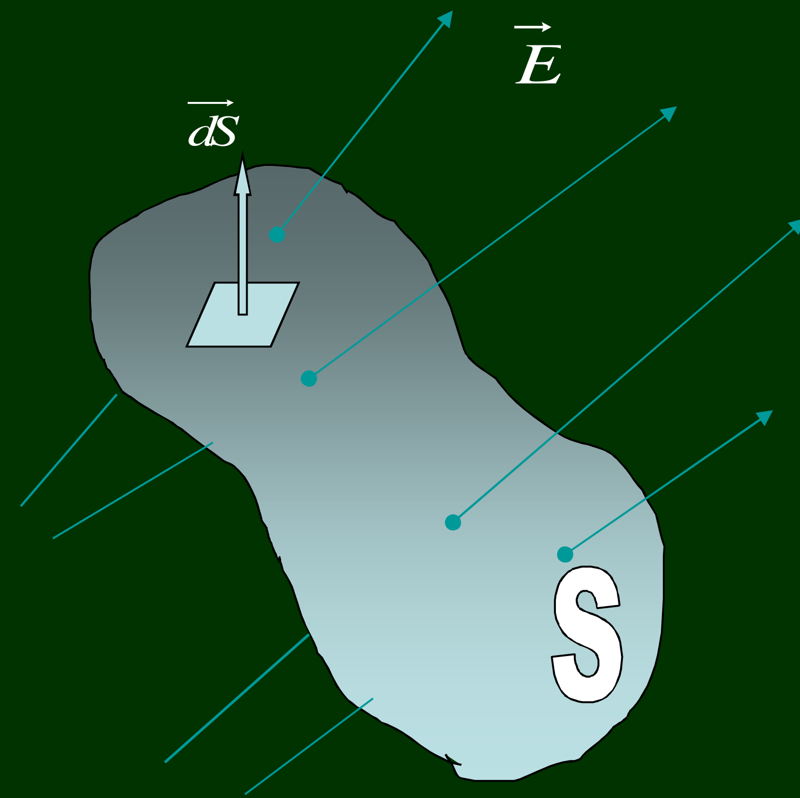
$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$$

…dobro opisuje za tačkasto naelektrisanje.





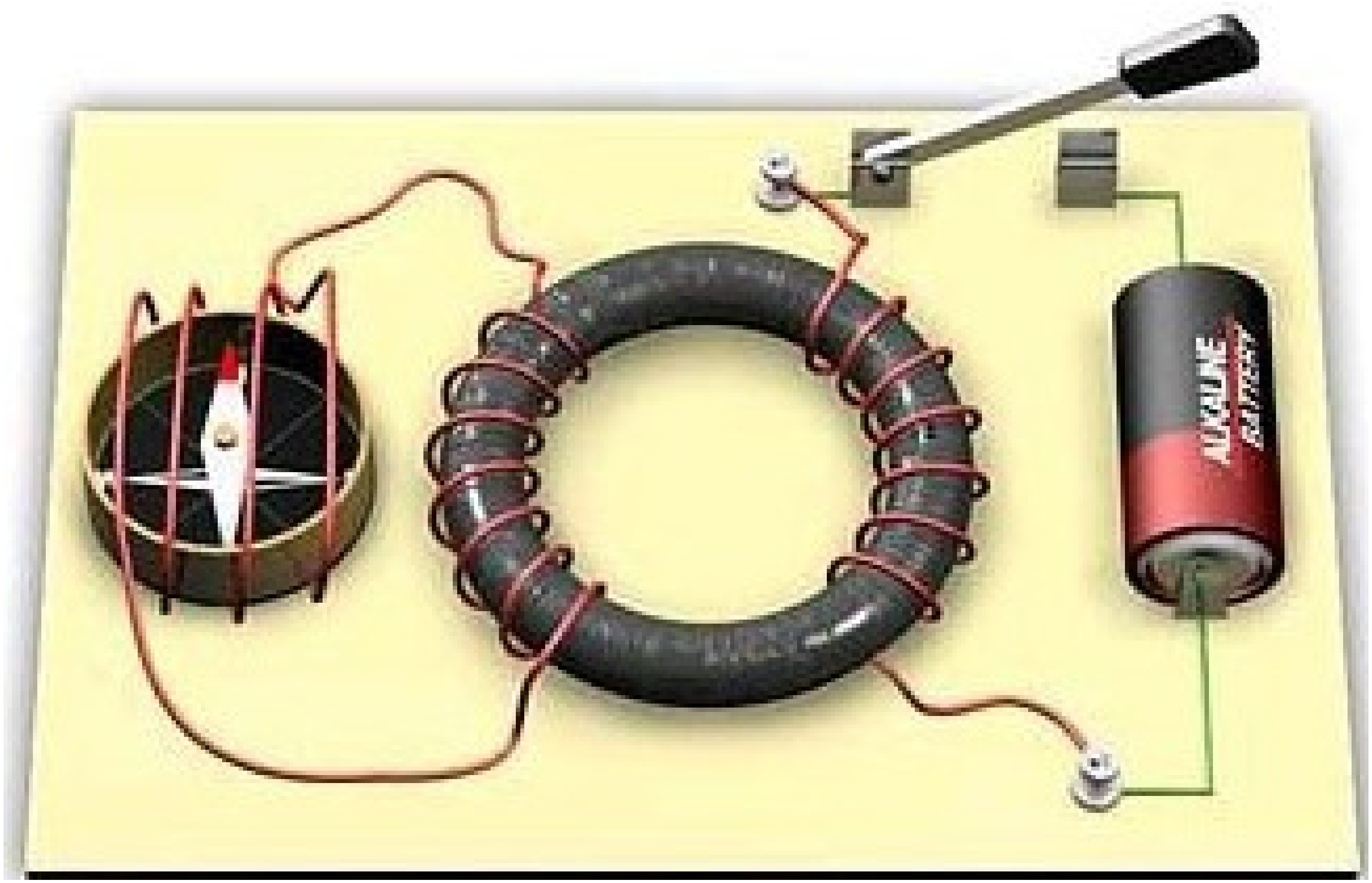




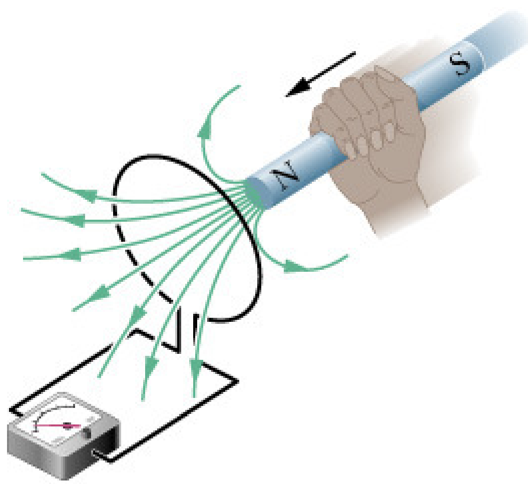
$$\oiint_S \vec{B} \cdot d\vec{S} = 0$$

$$\oiint_S \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0} \iiint_V \rho \cdot dV$$

III. Faradejev zakon indukcije

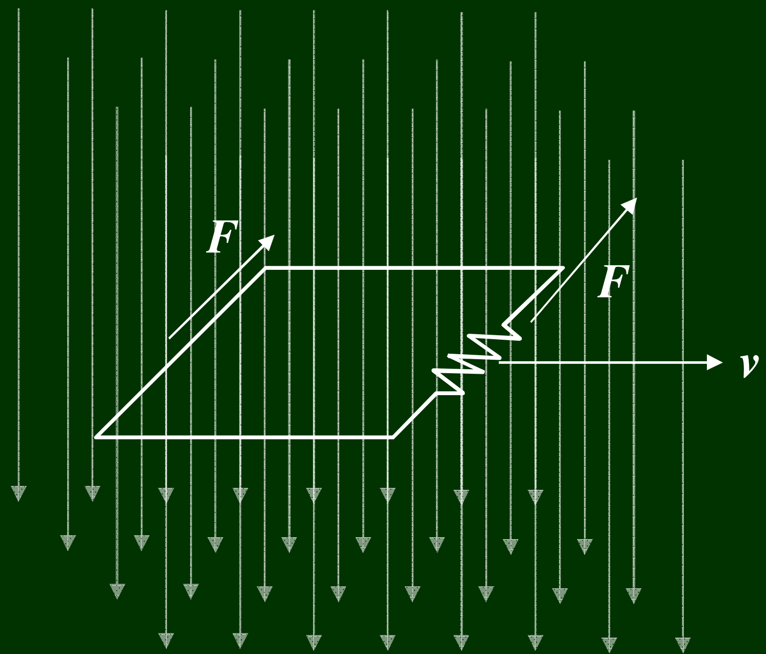


- **Promenljivi magnetni fluks generiše elektromotornu silu ems**
- **Ili, promenljivo B-polje generiše E-polje**
- **Promena magnetnog fluksa je potrebna**



$$\varepsilon = -\frac{d\Phi}{dt}$$

Sila koja deluje na provodne elektrone:

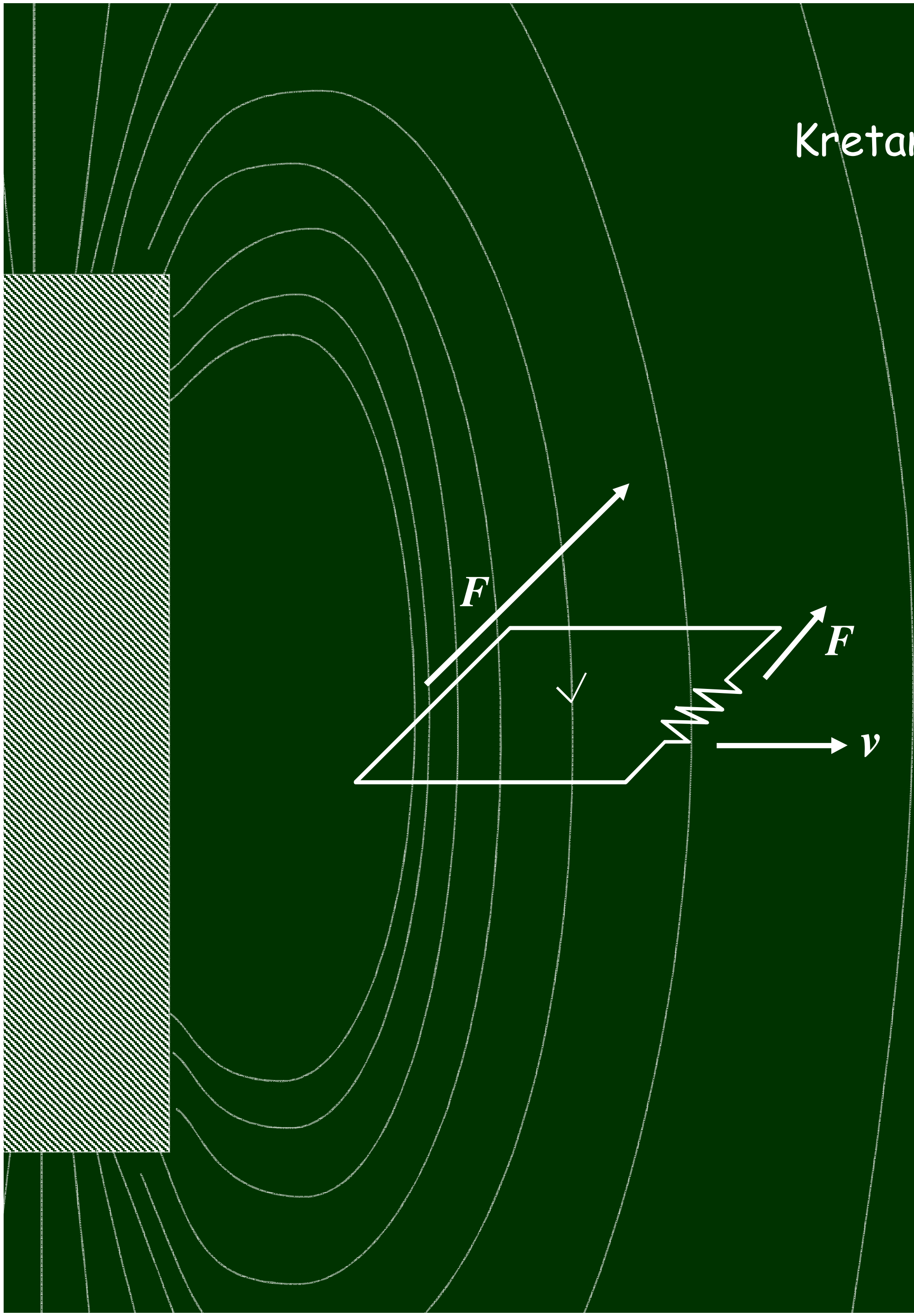



Faraday

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

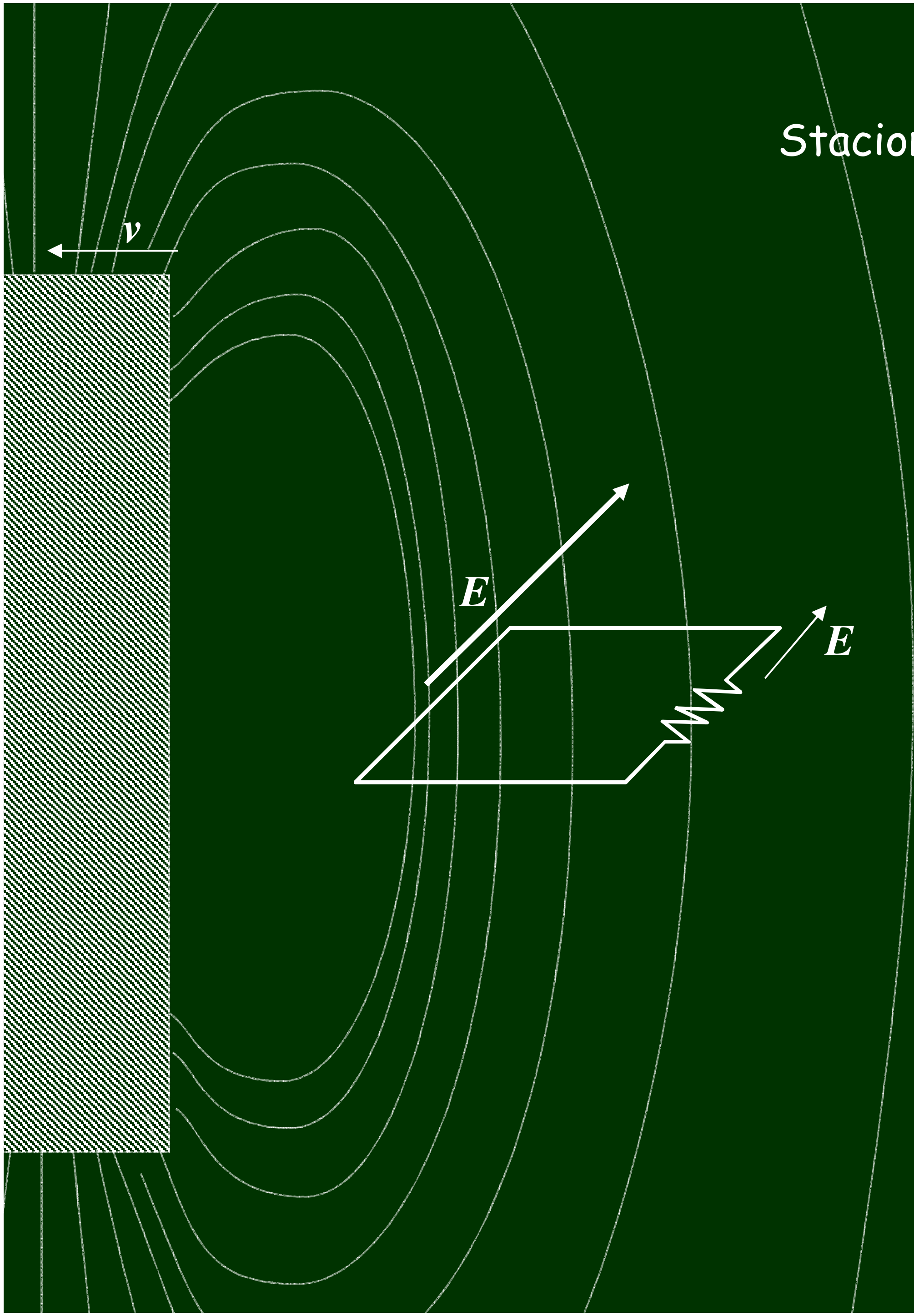
Nestankom sile: $ems = 0$

Kretanje namotaja u promenljivom ***B*** polju



$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Sila se ne poništava: $ems \neq 0$



Stacionarni kalem sa pokretnim $\boldsymbol{B}$ izvorom:

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

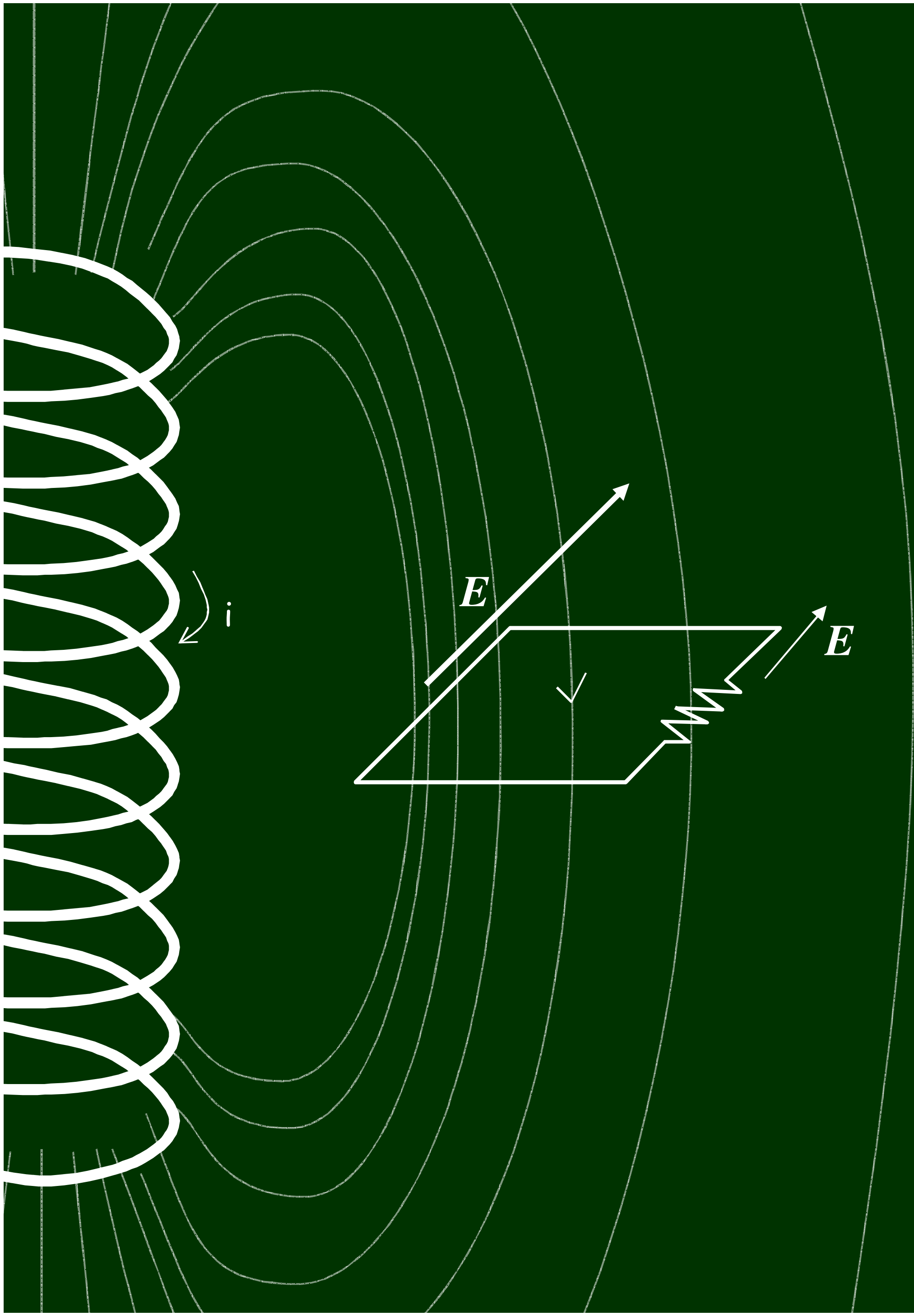
$$\vec{v} = 0$$

Ali mi još imamo *ems* …

Ostaje samo:

$$\vec{F} = q\vec{E}$$

Električno polje mora biti kreirano!



Stacionarni kalem i izvor $\boldsymbol{B}$ polja, ali se polje $\boldsymbol{B}$ povećava:

$$ems \neq 0$$

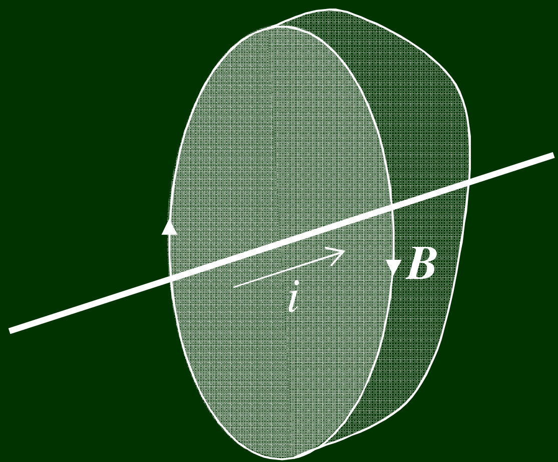
Uopšteno:

$$ems = -\frac{\partial \Phi_B}{\partial t}$$

$$\oint_C \vec{E} \cdot d\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

Faradejev zakon
(integralna forma)

IV. Amperov zakon



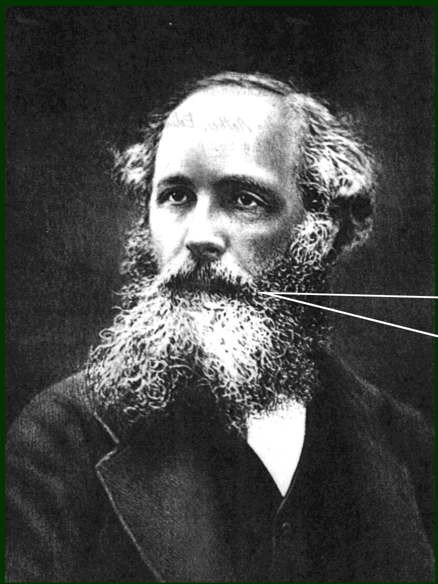
$$\oint_C \vec{\mathbf{B}} \cdot d\vec{l} = \mu_0 i_{zatvorena}$$

Oopštenije:

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \iint_S \vec{J} \cdot d\vec{S}$$

J = gustina struje

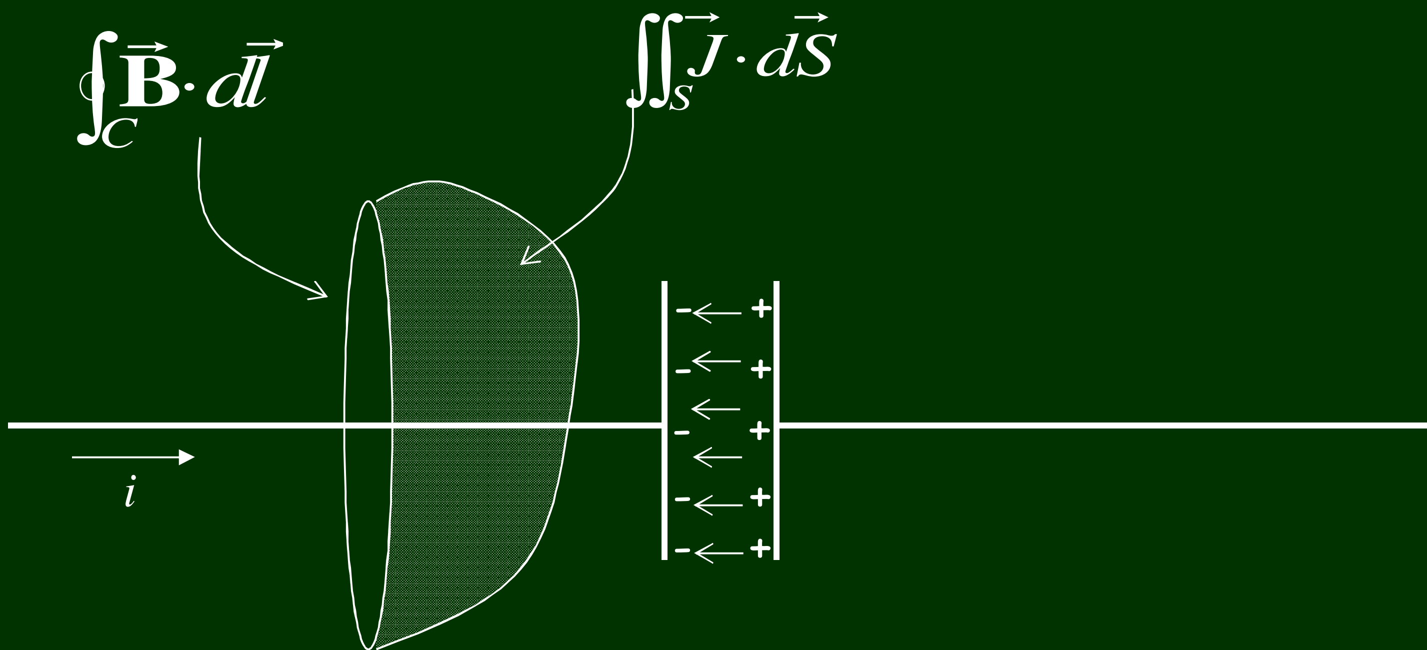
Ampere



Maxwell

"Nešto nedostaje..."

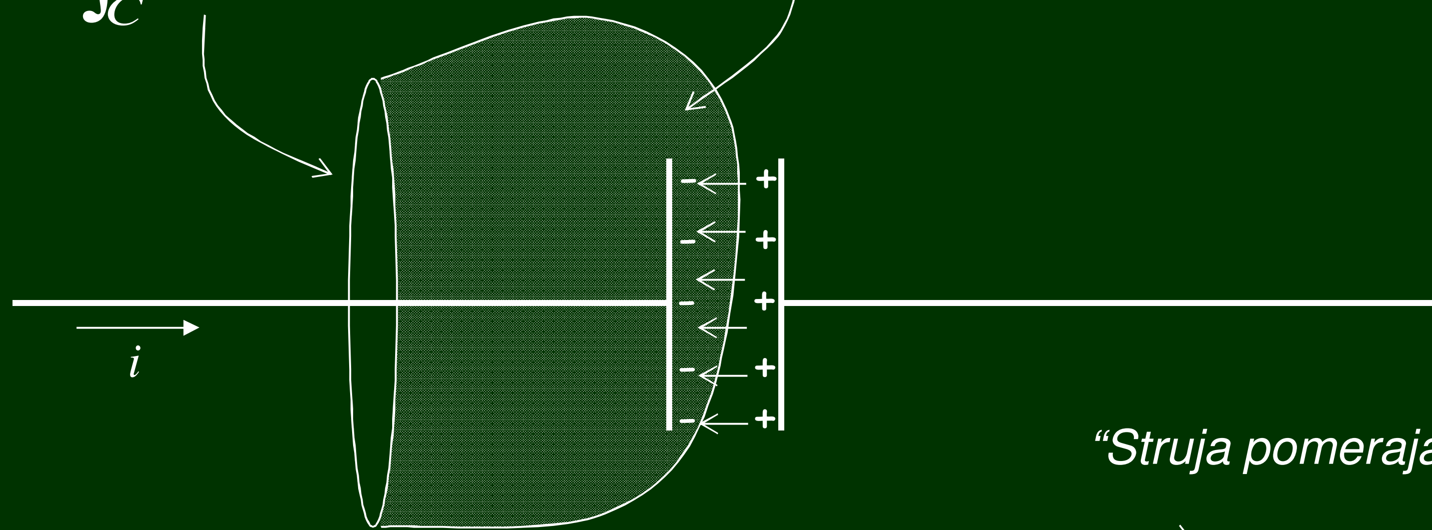
Punjenje kondenzatora



Punjenje kondenzatora

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}}$$

$$\iint_S \vec{J} \cdot d\vec{S}$$



"Struja pomeraja"

Maxwell: "...promena električnog polja u kondenzatoru je takođe struja."

$$\oint_C \vec{B} \cdot d\vec{l} = \mu \iint_S \left(\vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t} \right) \cdot d\vec{S}$$

MAKSVELOVE JEDNAČINE

(INTEGRALNA FORMA)

$$\oint_C \vec{E} \cdot d\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$\oint_C \vec{B} \cdot d\vec{l} = \mu \iint_S \left(\vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t} \right) \cdot d\vec{S}$$

$$\oint\!\!\oint_S \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0} \iiint_V \rho \cdot dV$$

$$\oint\!\!\oint_S \vec{B} \cdot d\vec{S} = 0$$

MAKSVELOVE JEDNAČINE U VAKUUMU
(INTEGRALNA FORMA)

$$\oint_C \vec{E} \cdot d\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$\oint_C \vec{B} \cdot d\vec{l} = \varepsilon_0 \mu_0 \iint_S \frac{\partial \vec{E}}{\partial t} \cdot d\vec{S}$$

$$\oiint_S \vec{E} \cdot d\vec{S} = 0$$

$$\oiint_S \vec{B} \cdot d\vec{S} = 0$$

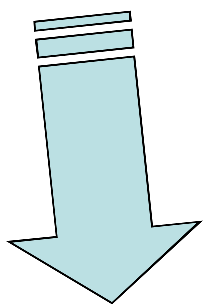
Maksvelovske jednačine u diferencijalnom obliku

Gausova teorema za divergenciju

$$\oiint_S \vec{F} \cdot \overrightarrow{dS} = \iiint_V \nabla \cdot \vec{F} dV$$

Stoksova teorema za rotor

$$\oint_L \vec{F} \cdot \overrightarrow{dl} = \iint_S \nabla \times \vec{F} \cdot d\vec{S}$$



$$\oint_L \vec{E} \cdot \vec{dl} = \iint_S \nabla \times \vec{E} \cdot d\vec{S}$$

$$\iint \nabla \times \vec{E} \cdot d\vec{S} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu\left(\vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

$$\oiint_S \vec{E} \cdot d\vec{S} = \iiint_V \nabla \cdot \vec{E} dV \qquad \iiint \nabla \cdot \vec{E} dV = \frac{1}{\varepsilon} \iiint \rho dV$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon} \qquad \nabla \cdot \vec{B} = 0$$

Elektromagnetni talasi

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \qquad \vec{E} = \frac{\vec{D}}{\varepsilon_0} - \frac{\vec{P}}{\varepsilon_0} \qquad \vec{D} = \varepsilon \vec{E} \qquad \vec{B} = \mu_0 \vec{H} + \vec{M} \qquad \vec{H} = \mu^{-1} \vec{B} \qquad \vec{J} = \sigma \vec{E}$$

$$\boxed{\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}} \qquad \boxed{\nabla \cdot \vec{B} = 0} \qquad \boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \qquad \boxed{\nabla \times \vec{B} = \mu\sigma \vec{E} + \mu\varepsilon \frac{\partial \vec{E}}{\partial t}}$$

$$\nabla \times \left(\nabla \times \vec{B}\right) = \mu\sigma\left(\nabla \times \vec{E}\right) + \mu\varepsilon \frac{\partial}{\partial t}\left(\nabla \times \vec{E}\right) \qquad \nabla \times \left(\nabla \times \vec{B}\right) = -\mu\sigma \frac{\partial \vec{B}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\nabla \times (\nabla \times) = \nabla(\nabla \cdot) - \nabla^2 \qquad \nabla \times \left(\nabla \times \vec{B}\right) = \nabla\left(\nabla \cdot \vec{B}\right) - \nabla^2 \vec{B} \qquad (\nabla \cdot \nabla)\vec{B} = \nabla^2 \vec{B} = \frac{\partial^2 \vec{B}}{\partial x^2} + \frac{\partial^2 \vec{B}}{\partial y^2} + \frac{\partial^2 \vec{B}}{\partial z^2}$$

$$\nabla^2 \vec{B} = \mu\varepsilon \frac{\partial^2 \vec{B}}{\partial t^2} + \mu\sigma \frac{\partial \vec{B}}{\partial t} = 0 \qquad \nabla \times \left(\nabla \times \vec{E}\right) = -\frac{\partial}{\partial t}\left(\nabla \times \vec{B}\right) \qquad \nabla \times \left(\nabla \times \vec{E}\right) = -\mu\varepsilon \frac{\partial \vec{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} - \mu\rho \frac{\partial \vec{E}}{\partial t} = \nabla(\rho / \varepsilon) \qquad \nabla\left(\nabla \cdot \vec{E}\right) = \nabla(\rho / \varepsilon) \qquad \nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} - \mu\varepsilon \frac{\partial \vec{E}}{\partial t} = 0$$

$$\boxed{\nabla^2 \vec{B} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0} \qquad \boxed{\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0}$$

$$v = 1/\sqrt{\varepsilon_0 \mu_0}$$

$$\varepsilon_0 \mu_0 = \left(8.85\times10^{-12}\, s^2 \cdot C^2 / m^3 \cdot kg\right)\left(4\pi\times10^{-7}\, m \cdot kg / C^2\right) = 11.12\times10^{-18}\, s^2 / m^2.$$

$$v = 1/\sqrt{\varepsilon_0 \mu_0} \approx 3\times10^8\, m/s.$$

$$c = 2.99792458\times10^8\, m/s.$$

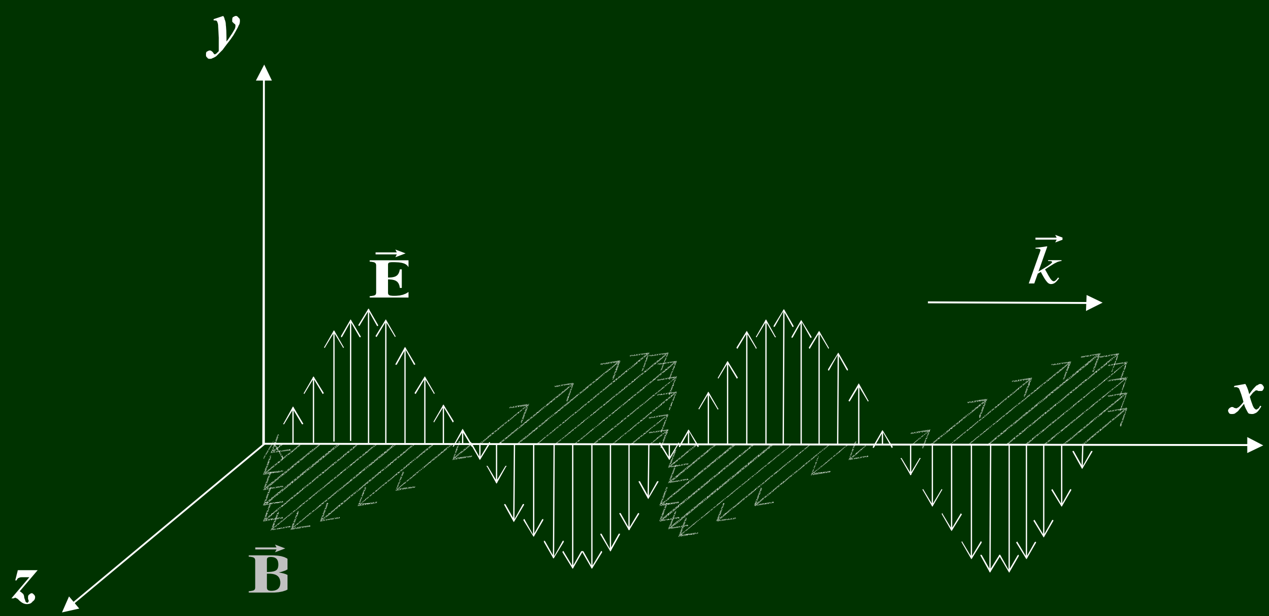
Transverzalni talsi

$$\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \qquad \vec{E} = \vec{E}(x,t) \qquad \frac{\partial E_x}{\partial x} = 0 \qquad \vec{E} = \vec{j} E_y(x,t)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\partial B_z}{\partial t} \qquad \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \qquad E_y(x,t) = E_{0y} \cos[\omega(t - x/c) + \varphi_0]$$

$$B_z = -\int \frac{\partial E_y}{\partial x} dt \qquad B_z = -\frac{E_{0y}\omega}{c} \int \sin[\omega(t - x/c) + \varphi_0] dt \qquad B_z(x,t) = \frac{1}{c} E_{y0} \cos[\omega(t - x/c) + \varphi_0]$$

$$E_y = cB_z$$



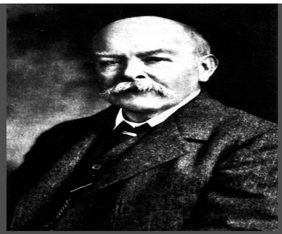
$$B_{0z} = \frac{E_{0y}}{c}$$

E_y, B_z su u fazi

$$\vec{\mathbf{E}} \perp \vec{\mathbf{B}} \perp \vec{k}$$

Ovo je ravanski talas kao rešenje EM talasne jednačine.

Energija i impuls
Pointigov (Poynting) vektor



John Henry Poynting

Gustina energije (J/m^3) u elektrostatičkom polju:

$$u_E = \frac{\varepsilon_0}{2} E^2$$

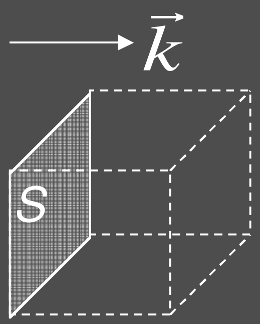
Gustina energije (J/m^3) u magnetnostatičkom polju:

$$u_B = \frac{1}{2\mu_0} B^2$$

$$u = u_E + u_B = \varepsilon_0 E^2 = \frac{1}{\mu_0} B^2$$

Brzina transporta energije: snaga (W)

$$P = \frac{energija}{\Delta t} = \frac{u\Delta V}{\Delta t} = \frac{uSc\Delta t}{\Delta t}$$

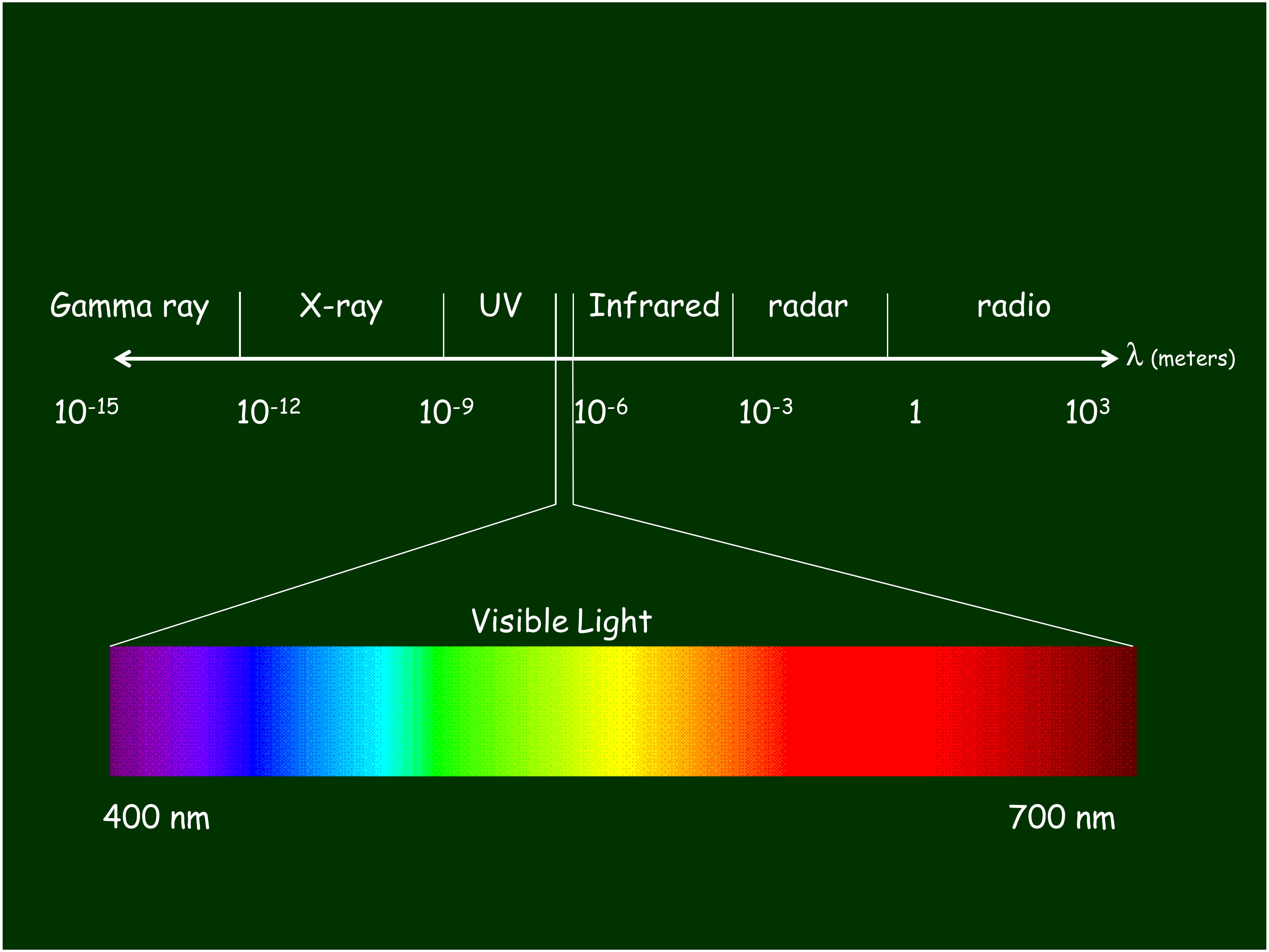


$c\Delta t$

Snaga po jedinici površine (W/m^2):

$$P = \frac{uc\Delta t S}{\Delta t S} = uc \quad P = \frac{1}{\mu_0} EB$$

$$\vec{P} = c^2 \varepsilon_0 \vec{E} \times \vec{B}$$


Gamma ray
X-ray
UV
Infrared
radar
radio
λ (meters)
10-15
10-12
10-9
10-6
10-3
1
103
Visible Light
400 nm
700 nm